Solving Continuous Decay Problems when given a half-life.

These techniques give you the most accurate answers for problems involving population growth, radioactive decay or anywhere you find the word “continuous” or “continuously.”

1. Begin with the formula: \( a = pe^{rt} \)

   Where,
   - \( a \) = amount at time \( t \)
   - \( p \) = principal (starting amount)
   - \( e \) = “Euler’s number” a constant 2.718…..
   - \( r \) = rate in decimal form (i.e. 9% = 0.09)
   - \( t \) = time (in years, hours, days…)

2. Find the rate:

   No matter what your starting amount is, a half-life is the amount of time it takes until half of that amount remains. So you can leave out the starting amount (\( p \)) and simply set the left side of the equation to 0.5. 2 examples:

   A) half-life is 6 hours.
   
   Your equation is \( 0.5 = e^{6r} \)

   B) half-life is 10,000 years

   Your equation is \( 0.5 = e^{10,000r} \)

   Solve for \( r \) using natural logs:

   \[
   \begin{align*}
   0.5 &= e^{6r} \\
   \ln 0.5 &= \ln e^{6r} \\
   \ln 0.5 &= 6r \\
   \frac{\ln 0.5}{6} &= r \\
   -0.1155 &= r
   \end{align*}
   \]

   \[
   \begin{align*}
   0.5 &= e^{10,000r} \\
   \ln 0.5 &= \ln e^{10,000r} \\
   \ln 0.5 &= 10,000r \\
   \frac{\ln 0.5}{10,000} &= r \\
   -0.0000693 &= r
   \end{align*}
   \]

3. Plug the rate in for \( r \) and use the resulting equation for any further questions:

   A) \( a = pe^{-0.1155t} \)

   B) \( a = pe^{-0.0000693t} \)
4. Example questions:

A) A medicine has a half-life of 6 hours. If a patient is given 600 mg at noon, at what time do they have 100 mg remaining in their bloodstream?

\[ a = pe^{-0.1155t} \]
\[ 100 = 600e^{-0.1155t} \]
\[ 0.16 = e^{-0.1155t} \]
\[ \ln 0.16 = \ln e^{-0.1155t} \]
\[ \ln 0.16 = -0.1155t \cdot \ln e \]
\[ \ln 0.16 = -0.1155t \]
\[ \frac{\ln 0.16}{-0.1155} = t \]
\[ 15.5 = t \]

Answer is 15.5 hours after noon; 3:30 AM the next morning.

B) Cobalt 121 is a radioactive substance with a half-life of 10,000 years. A geologist finds a deposit of Cobalt 121 which weighs 1.5 grams. He estimates the original deposit contained 20 grams of Cobalt 121. How old is the deposit?

\[ a = pe^{-0.0000693t} \]
\[ 1.5 = 20e^{-0.0000693t} \]
\[ 0.075 = e^{-0.0000693t} \]
\[ \ln 0.075 = \ln e^{-0.0000693t} \]
\[ \ln 0.075 = -0.0000693t \cdot \ln e \]
\[ \ln 0.075 = -0.0000693t \]
\[ \frac{\ln 0.075}{-0.0000693} = t \]
\[ 37,377.59 = t \]

The deposit of Cobalt 121 is 37,377 years old.

5. Other examples involving continuous growth:

a) You put $2000 into a savings account that pays 6% interest, compounded continuously. How much will be in the account in 13 years?

\[ a = 2000e^{0.06 \cdot 13} \]
\[ a = 2000e^{0.0613} \]
\[ a = 4,362.94 \]

The account will hold $4,362.94 in 13 years

b) In how long will the account be worth $100,000?

\[ 100,000 = 2000e^{0.06t} \]
\[ 50 = e^{0.06t} \]
\[ \ln 50 = \ln e^{0.06t} \]
\[ \ln 50 = 0.06t \cdot \ln e \]
\[ \ln 50 = 0.06t \]
\[ \frac{\ln 50}{0.06} = t \]
\[ 65.2 = t \]

The account reaches $100,000 in approx. 65 years.